Efficient Optimization of Aircraft Structures with a Large Number of Design Variables

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The number of variables within structural optimization procedures must be reduced to achieve acceptable low computer time. A method is presented that reduces the number of variables for a large finite element wing-box structure by representing the thickness distribution of the wing-box elements using mathematical functions. The control points of these mathematical functions are handled within the optimizer, thus greatly reducing the number of variables. The advantage of this procedure is the variational freedom achieved using functions to represent the thickness distribution of the structural elements is virtually unchanged. Element thickness of a wing box consisting of 430 finite elements is optimized using the proposed method. The number of variables is reduced to 57, and computation time is decreased by a factor of 4.2. Results indicate a very good optimization behavior. Furthermore, this method is used with the classical variable slaving or coupling method to yield an additional reduction in computation time.

Introduction

OPTIMIZATION of aircraft and spacecraft structures is driven by the need for accuracy and low computation time. To meet the required accuracy, stresses and strains of the structures are analyzed using finite element methods even in predesign phases.[1] However, it is impossible to match the demand for low computation time when treating all finite element thicknesses as variables for optimization of thickness distribution (sizing process) or all node point locations as variables for optimization of shape. Therefore, the number of variables must be reduced. A standard procedure for reducing the number of variables is to couple the variables of a set of elements, for example, the elements of the wing-box plate between two ribs of the wing, and treat this condensed set as one variable.[2] This method is known as variable slaving or variable coupling. When a constant thickness is required, variable slaving may be used to meet manufacturing constraints, but it cannot lead to a low-mass optimum solution because the procedure cuts feasible parts of the design space that are useful for the sizing process.

Current procedures for shape optimization make use of mathematical functions for coupling of node point locations while treating only the coefficients of the function as variables in the optimizer.[3,4] The node point locations of the finite element grid are then extrapolated from this function. Thus, the number of variables is reduced, and the optimization process is smoothed without reducing the variational freedom.

The idea presented in this paper uses a similar method of variable reduction for thickness optimization of large wing-box structures that has proven to give good results.[5] The expected advantage of using this procedure is to have little or no change in variational freedom when mathematical functions which represent the optimum thickness distribution are implemented. In this case the full feasible design space is available for optimization. Figure 1 gives an overview of how the computer time is decreased when the variable number is reduced. Reducing the number of variables from 430 (a moderate finite element set) to 57 decreases computation time by a factor of 4.2, and reducing the number from 4300 (a large finite element set) to 57 decreases the computation time by a factor of 7.4.

Implementing Variable Reduction

The numerical optimization process consists of a preprocessor-analyzer-postprocessor-optimizer loop. A variable reduction process is introduced between the preprocessor and the optimizer (Fig. 2) because it is inefficient to handle the large set of the finite element analysis. Several methods are in use for variable set reduction. These methods are listed in order according to their reduction of the variational freedom of the design process: full variable set, mathematical functions, variable coupling or slaving, and disregard or fixing of variables.

Full variational freedom may be achieved by implementing correct mathematical functions, such as a cubic function when a cubic thickness distribution is the expected optimum. Using a square function would then reduce the variational freedom and would not allow a minimum value to be found. Other methods, such as the variable slaving or, as a worst case, the fixing of variables, reduce the variational freedom. This reduction in variational freedom becomes important when additional mass reduction is required.

The mathematical formulation for using functions to reduce the number of variables is described as follows. An objective function $F(x)$ is minimized with respect to a set of constraints $g(x)$, $h(x)$ for a given variable set x and a given number of variables nv:

$$\min F(x)$$

$$g(x) \leq 0$$

$$h(x) = 0$$

$$x = [x_1, \ldots, x_{nv}]^T$$

Instead of treating the finite element thicknesses $t_1, \ldots, t_{ne}$ (ne = number of elements) as variables within the optimizer such that

$$x = [t_1, \ldots, t_{ne}]^T$$

the control point values $v_1, \ldots, v_{nc}$ (nc = number of control points) of a mathematical function $G(v_1, \ldots, v_{nc}, c_1, \ldots, c_{nc})$, with control point locations $c_1, \ldots, c_{nc}$, are treated as

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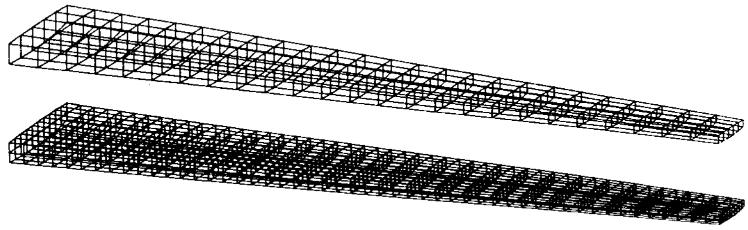

Finite elements	430	430	4300	4300
Variables	430	57	4300	57
Computer time	111 min	27 min	1473 min	199 min
Factor	4.2		7.4	

Times are given for 10 iteration cycles on CDC 180-860

Fig. 1 Comparison of full and reduced variable set optimizations.

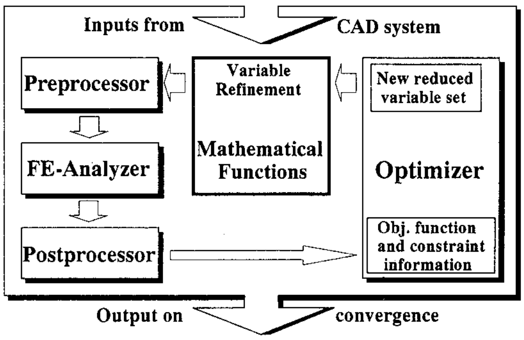

Fig. 2 Variable reduction within the optimization process.

variables. This results in the following variable vector within the optimization process:

$$x = [v_1, \ldots, v_{nc}]^T$$

This function is optimized with respect to the unchanged constraint set. The thicknesses of the finite elements are then extrapolated from the function (variable refinement process). Thus, a large set of element thicknesses is controlled by a single function which greatly reduces the number of variables in the optimization process. For example, a square polynomial function, shown in Fig. 3, needs only three coefficients or three control point values to be described completely. From this square function, an arbitrary number of finite element thicknesses may be controlled, as shown in the lower sketch of the figure. In addition, the use of higher-order functions and two-dimensional functions (useful for plate thickness optimization) is straightforward.

Numerical Examples

The described method is implemented for optimization of a wing-box structure of a twinjet aircraft. Four different formulations of the optimization problem with respect to the variable reduction process are investigated: 1) the full unreduced variable set, 2) a variable set reduced using mathematical functions, 3) a variable set reduced by variable slaving, and 4) a variable set reduced by combined use of mathematical functions and variable slaving.

The first example is a nonreduced variable set optimization process computed for reference purposes. All element thicknesses are treated as variables, and the corresponding stresses are treated as constraints. This method yields 430 variables for the complete wing box.

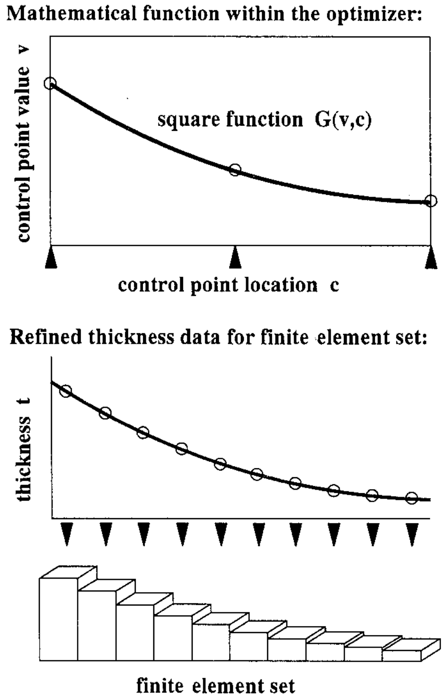

Fig. 3 Principle of using mathematical functions.

In example 2, mathematical functions are used to reduce the number of variables for the optimization. The variables within the optimization process consist of the control points of two-dimensional mathematical functions describing the thickness distribution of the upper and the lower wing-box panels and the control points of the one-dimensional functions of the wing-box ribs and spars. Finite element thicknesses for each rib are coupled, and all rib thicknesses are deduced from those one-dimensional functions. The overall optimizer model consists of 57 variables (Fig. 4). The single load from the engine and the wing kink make it necessary to use two mathematical functions in the spanwise direction for variable reduction (one for the range from the wing tip to the wing kink and one from the kink to the junction with the center wing box) in order to describe the expected discontinuity within the thickness distribution. Two-dimensional square/cubic mathematical functions are used to correctly describe the thickness distributions of the wing-box panels with full variational freedom. This corresponds with theoretical thickness calculations for a clamped linear tapered box structure. Spars are treated by two square functions each, which gives enough variational freedom to match the design stress load in all elements. Ribs are treated with two square functions.

For comparison, example 3 uses the classical method of variable reduction, variable slaving. Element thicknesses for all elements on the upper and on the lower wing-box plate in between two ribs are coupled and treated as one free variable. The number of variables for the spars and the ribs are not reduced, resulting in a total of 225 variables.

A fourth example is formulated, using both the variable slaving procedure from example 3 and the variable reduction by use of mathematical functions from example 2. This scheme requires only two one-dimensional functions in the spanwise direction for each of the plates, ribs, and spars. The result is a total of only 25 variables for the complete wing-box model. The variable sets for the four different variable reduction formulations are shown in Fig. 5 for comparison.

Whereas optimizing the thickness of a structure for sizing (and therefore weight reduction) is a linear process, the constraints are nonlinear. The well-established penalty-function method is successfully used in conjunction with a conjugate-gradient procedure[6,7] for the nonlinear multivariate optimization. Plane finite elements are used for stress calculation. The overall model is presented in Fig. 4. The load case for the wing is the maximum g-load factor. Aerodynamic loads, wing-box mass and fuel weight, the mass and thrust of the engine, and the wheel-base mass are considered. An additional manufacturing constraint is taken into account by setting the minimum thickness to 1 mm. This is an important consideration for the wing-tip zones.

Results and Comparison

Reference for comparison is the optimization process with 430 variables, example 1, which requires 111 min total CPU time (Fig. 6). This yields a maximum stress difference on the upper panel of 5 N/mm^2, taking into consideration only the areas where the minimum thickness constraint is not active. The 300 N/mm^2 design stress level constraint is met, and the optimization procedure is shown to be efficient. With the reduced variable set of example 2, shown in Fig. 4, computer time is decreased to 27 min, a reduction by a factor of 4.2. The weight increase is only 1% and the maximum difference in stress is 10 N/mm^2. The global optimum, computed using the full variable set, was nearly reached in example 2, the variable reduction by mathematical functions. Figure 7 shows the isostress contours of both wing-box panels for examples 1 and 2. For clarity, only the upper wing-box panel data are presented. For the outer wing area, where the manufacturing constraint is active, almost no difference is seen. In the fuselage area, the stress distribution of the wing-box panel varies due to the continuous thickness increase computed by the interpolating functions. For the full variable set optimization, the thickness in this area is reduced due to the stiffness of the rear spar structure. The square function used in the chord direction for the reduction procedure cannot match the thickness change of the full variable set and yields average values.

For the chordwise coupled optimization, example 3, there is an 11% increase in mass compared to the global optimum and a maximum stress difference on the upper panel of 30 N/mm^2. Overall computer time is 41 min. Comparing the full variable set optimization of example 1 to the variable slaving procedure clearly shows the different optima that may be achieved and the weight increases. The chordwise constant reduced variable set optimization, example 4, leads to the same weight change as example 3, with a slight increase in the stress difference. Computer time was reduced to only 19 min. Thickness distributions, shown in Fig. 8, show the square behavior and the coupled thickness across the chord. The reduction in the variational freedom due to the coupled variables leads to much larger differences in the stress distribution compared to the full variable set. A comparison of the results of both chordwise coupled optimizations, examples 3 and 4, shows little difference in relative weight and maximum stress level difference, which leads to the conclusion of using mathematical functions for controlling the reduced variable sets of chordwise coupled thickness optimizations and for reducing the computation time with it.

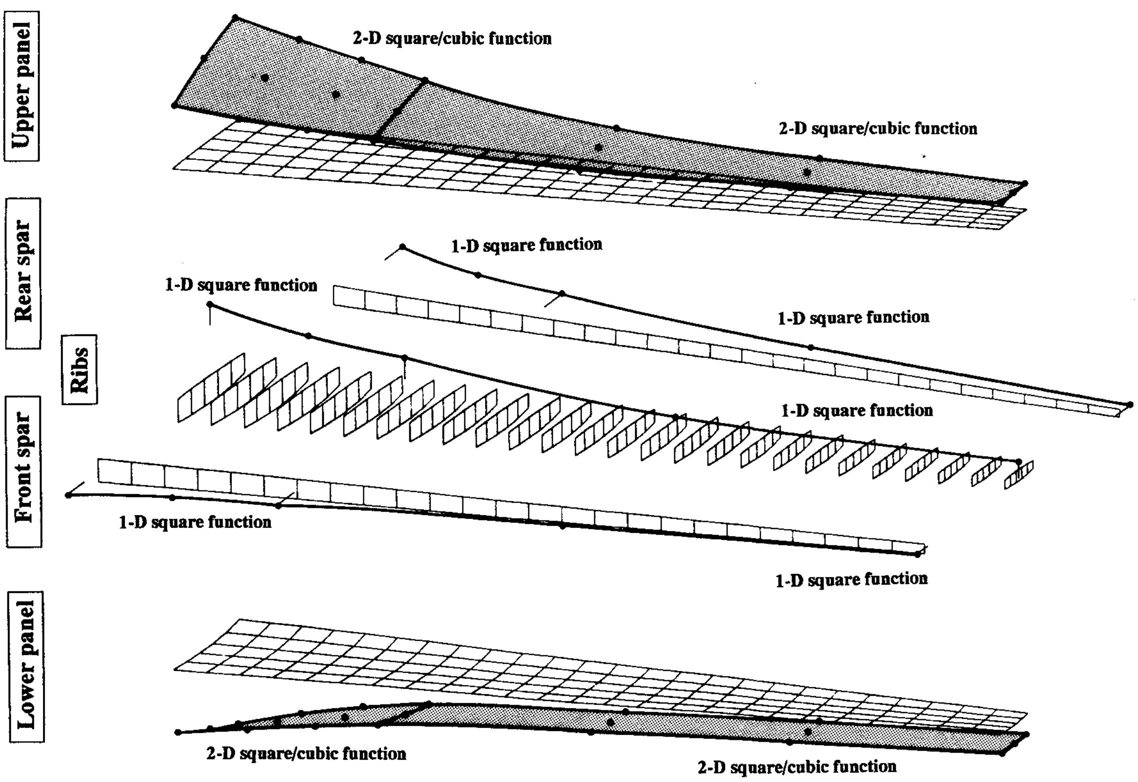

Fig. 4 Interpolating functions for thickness optimization of a twinjet wing box.

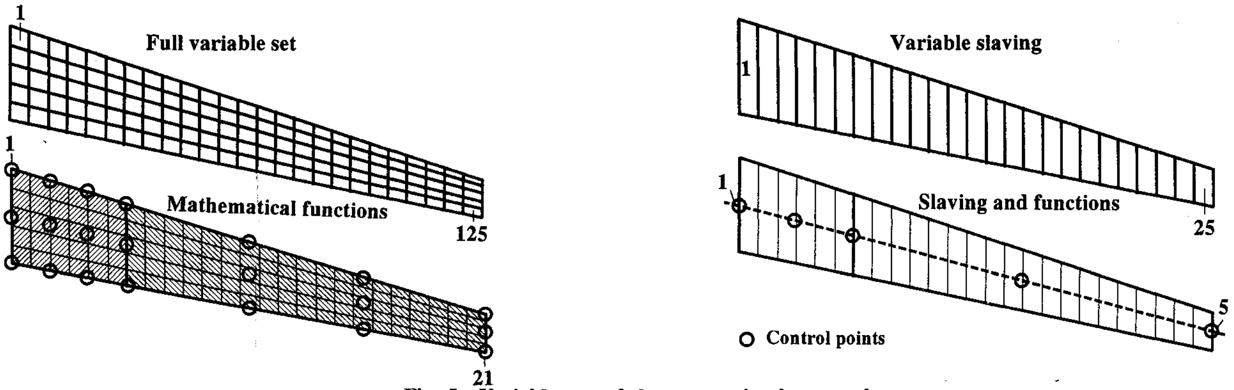

Fig. 5 Variable sets of the upper wing-box panel.

	Relative weight of complete wing box	Maximum difference of stress level on upper wing-box panel	Number of variables for upper wing-box panel	Computation time
Free variable set	100 %	5 N/mm^2	125	111 min
Reduced variable set	+ 1 %	10 N/mm^2	21	27 min
Chordwise constant variable set	+ 11 %	30 N/mm^2	25	41 min
Chordwise constant reduced variable set	+ 11 %	33 N/mm^2	5	19 min

10 iterations CDC 180-860

Fig. 6 Weight optimization of a wing box with variable reduction.

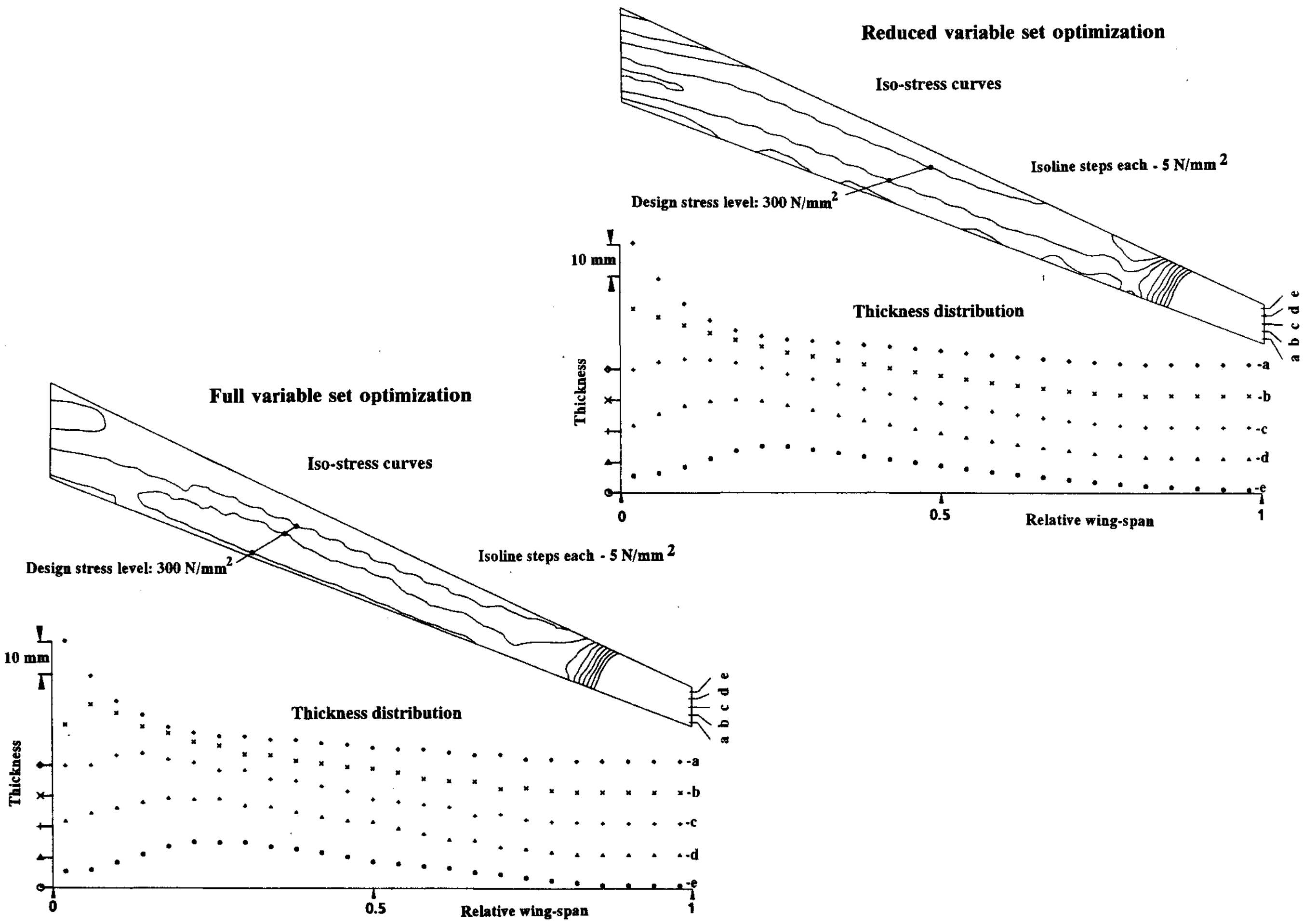

Fig. 7 Comparison of full and reduced variable set thickness optimizations.

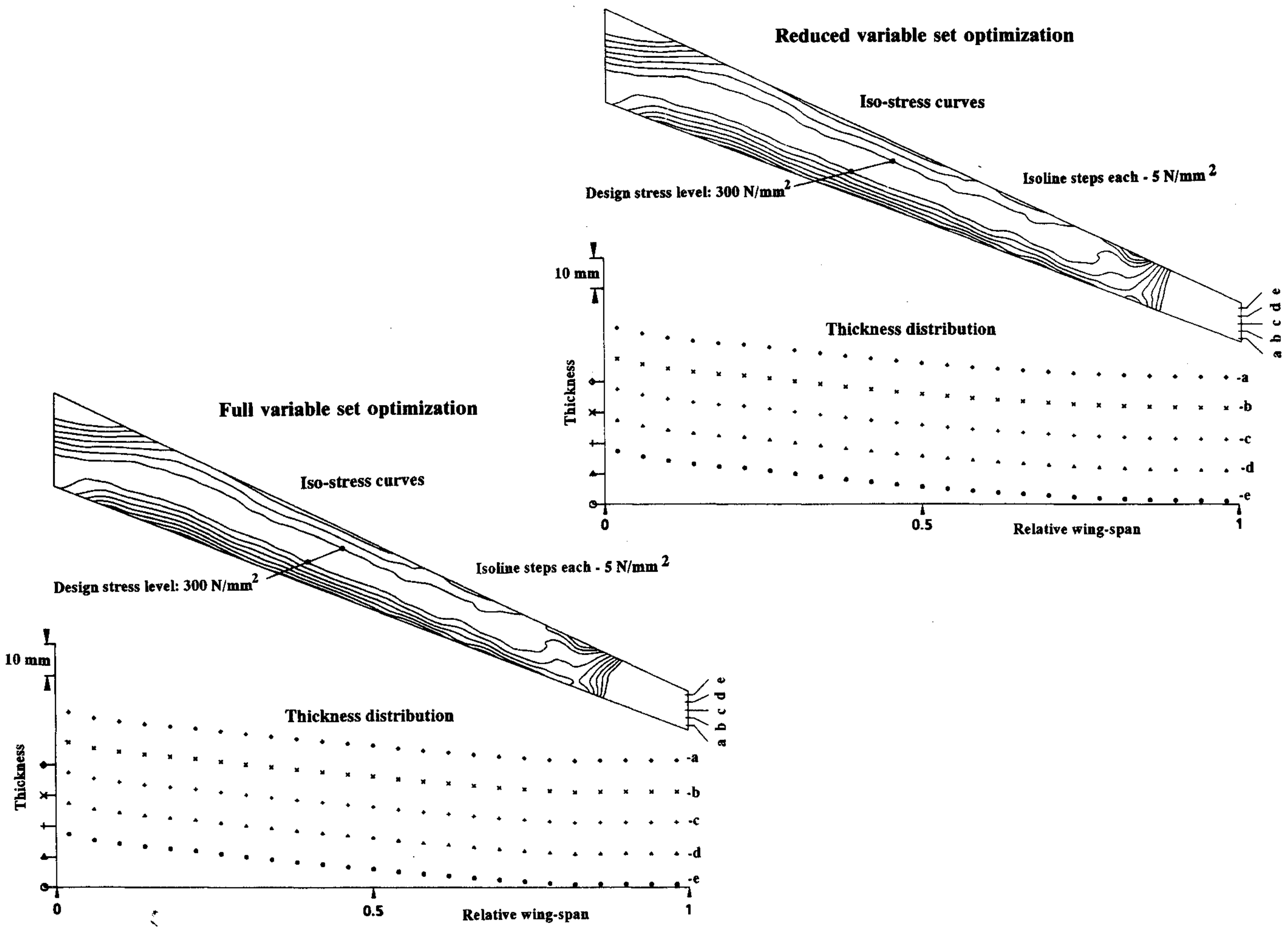

Fig. 8 Comparison of full and reduced variable set thickness optimizations with constant thickness over wing-box chord.

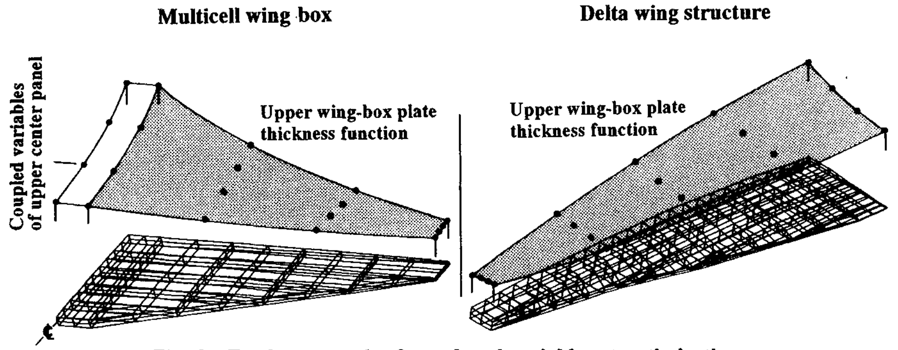

Fig. 9 Further examples for reduced variable set optimization.

Experiences

In general, variable reduction by use of functions is a very efficient method, but obtaining reliable results depends very much on the functions selected for the procedure. The following four recipes are useful guidelines for selecting functions for variable reduction.

1) Avoid using functions of higher order than necessary; this may lead to an unstable optimization procedure.

2) Compute the function from a set of control points instead of controlling the function by its coefficients; the control point method is much less sensitive and will ameliorate the optimization behavior.

3) The first optimization iteration should lead to a feasible initial variable vector; this avoids unstable behavior after the first variable change, due to very different search gradients.

4) Use several functions when inconsistent thickness distributions are to be expected, for example, because of single loads or geometry changes.

Other Applications

Figure 9 gives an overview of two other examples, which are interesting for structural optimization. One is a multicell wing box, such as that used in fighter aircraft. To represent the thickness distribution of the upper wing-box panel, a two-dimensional cubic/cubic mathematical function is used with coupled variables on the center wing-box panel. The lower wing-box panel may be treated in a similar manner, whereas the thickness distribution for the spars and ribs is represented by one-dimensional square functions, as was done for the twinjet wing box. The variables are reduced to 62 with 6 spars and 5 ribs.

Another example is a delta wing structure, which is used for hypersonic aircraft. The upper wing-box panel thickness distribution is represented by a two-dimensional function with fourth-order behavior in the chord direction and cubic behavior in the spanwise direction. This gives more flexibility for the thickness distribution in chord direction, which is necessary due to the long wing root and the expected local thickness differences. Spars and ribs are treated in the same manner as previous examples.

Conclusion

Optimization of a wing-box structure by variable reduction using mathematical functions yields results nearly identical to those achieved from a full variable set optimization. In addition, a large decrease in computer time, without the decrease in variational freedom seen when using the variable slaving method, is obtained. If the optimization results achieved from a variable set reduced by variable slaving are acceptable, the optimization achieved for a set reduced using a combination of the variable slaving method with the mathematical function formulation will yield virtually identical results and an additional decrease in computer time.

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References

[1] Fredriksson, B., "Computerized Methods for Analysis and Design of Aircraft Structures," *Proceedings of the 14th ICAS Congress,* Toulouse, France, AIAA, New York, 1984, ICAS 84-4.7.2.

[2] Wellen, H., "Anwendungen der rechnergestützten Strukturoptimierung bei der Auslegung von Flugzeugbautielen" ("Applications of the Computer Aided Structural Optimization to Aircraft Components"), DGLR-Fachausschußsitzung Festigkeit und Bauweisen (DGLR-Expert Meeting Strength and Structures), Neubiberg, Germany, 1984.

[3] Botkin, M. E., "Shape Optimization of Plate and Shell Structures," Technical Papers of the 22nd Structures, Structural Dynamics and Materials Conference, Atlanta, GA, April 1981, pp. 242–249.

[4] Bennet, J. A., and Botkin, M. E., "Structural Shape Optimization with Geometric Description and Adaptive Mesh Refinement," *AIAA Journal,* Vol. 23, March 1985, pp. 458–464.

[5] Walsh, J., "Application of Mathematical Optimization Procedures to a Structural Model of a Large Finite Element Wing," NASA TM-87597, Jan. 1986.

[6] Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design,* McGraw-Hill, New York, 1984.

[7] Haftka, R. T., and Kamat, M. P., *Elements of Structural Optimization,* Martinus Nijhoff, The Hague, the Netherlands, 1985.